A Discrete Optimization Approach to Airline Flight and Captain Scheduling

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ABSTRACT – The number of flights that occur every day around the world is extremely large. Each and every one of these flights both private and commercial need to be finely scheduled for many reasons including safety, timeliness, and operational efficiency. In the commercial world, scheduling not only the flights, but the captains to operate the flights is important to maximize profit. An approach to scheduling captains on flights is given here using discrete optimization methods as well as a post-optimality analysis is shown here. The results show a scheduling formulation method for scheduling pilots within the FAA guidelines.

1. Introduction

The airline industry is constantly trying to find ways to maximize profit, while minimizing costs to customers to increase patronage. These market pressures are acted against by requirements that are put forth for the safety of travelers, crew, and public by laws and regulations. Such restrictions include maintenance protocols specifying the regularity that planes must be inspected as well as guidance on pilot flight length times and work hours. This second group is what we will focus on here. The demand for flights is generated by the airlines for various sophisticated methods. By looking at historical data and forecasted data, the companies can generate a list of flights that need to be flown, or at least, can be flown at a profit. Finding captains for the flight then incurs an addition cost of operation. Here we will take a given list of flights and requirements, formulate into a binary linear model, and give a general solution framework for understanding this problem.

II. Literature Review

Linear programming has been famously applied to scheduling problems throughout its history. Applications include nurse scheduling (Brigitte Jaumard, 1998), production scheduling (Bowman, 1956), and even smart home appliances. (Kin Cheon Sou, 2011). There is a large corpus of work in this area. With particular regards to flight/flight captain scheduling, there is work done with scheduling plane landings with mixed integer zero-one formulation that created both an optimal solution and heuristic approach (J. E. Beasley, 2000) as well as air freight scheduling where schedules must be determined in advance (Julia L. Higle, 1996). A formulation is given here which takes into account limiting factors such as the flying hours an aircraft is limited by, as well as the routes that pilots are allowed to take. For insights on crew scheduling, there has been work done to look at the ways flights and crews can be scheduled in conjunction to minimize costs, such as in (Oliver Weide, 2010) However, the strategy applied here is an iterative solution. Decomposition methods have also been investigated to simultaneously schedule both flights and crews (Jean-Francois Cordeau, 2001)

III. Statement of the Problem

The problem we are trying to solve is the gap between a list of flights that an airline has deemed required/desirable/profitable to fly in conjunction with a database of pilots and their hourly rates of employment and a prescriptive assignment that minimizes the cost for the airline. That is, given a list of flights and pilots, we seek to provide a binary integer linear programming framework for assigning the pilots in an optimal fashion.

IV. Assumptions

To aid the effectiveness of this work, the following assumptions are made. First, we will use “hub scheduling” in which an airline combines all pilots that share a particular home location into a single list. This ensures that the flights resemble a circular path and return at the home. Second, to ensure linearity is not violated, we will assume that a pilot must be returned to fly again. This means that flights will not overlap. At larger scale this is not viable or applicable but for our short-term modeling, we will still be able to gain insight based off of this. We will also assume that there are no delays. This can be justified by assuming that any delay time is put into the model as part of the flight time. A simple refinement procedure of padding times with samples from delay distributions can correct this quickly.

V. Notation and Mathematic Formulation

To that end, here we will investigate a linear programming model which takes as its input a list of flights, with information, and a list of pilots, also with information, and outputs an assignment prescription. We will utilize binary linear optimization; the decision variables will be entries in a P by F matrix, where P is the number of pilots and F is the number of flights:

Table 1: An example table of Flights with ID number, Flight Name, Departure Time, Arrival Time, and Flight Time

ID	Name	Depart	Arrive	Time
1	F6	20	27	7
2	F2	26	31	5
3	F8	65	68	3

Table 2: An Example of Pilots with Name/ID, Hourly Rate

Name	Rate
P1	$45
P2	$53

Table 3: An example of an Assignment Matrix. Pilot 1 will be on flights F1 and F3, Pilot 2 on F3

	F1	F2	F3
P1	1	0	1
P2	0	1	0

Formulating this into an LP problem we have:

$$MIN\ Cost = \sum_{p=1}^{P} \sum_{f=1}^{F} t_f r_p x_{p,f}$$

Subject to:

$$(1) \quad \forall f \, , \sum_{p=1}^{P} x_{p,f} \geq 1$$

$$(2) \quad \forall p \, , \sum_{f=1}^{F} x_{p,f} t_f \leq 60$$

$$(3) \quad \forall d \, , \forall p, \qquad \sum_{f=1}^{F} x_{p,f} t_f d \leq 8$$

$$(4) \qquad x_{p,f} \; binary$$

Where the variable $x_{p,f}$ reflects in binary, if a pilot p will be on flight f, P is the number of pilots, F is the number of pilots, r_p is the hourly rate of pilot p, t_f is the time that flight f takes to fly, and d is a binary unit vector indicator reflecting that flight f is on day d.

These constraints come from basic scheduling principals of scheduling as well as FAA regulations. First consider (1) this guarantees that each flight has a captain, by requiring at least on decision variable is positive. It could be the case the there are two captains on a flight for the purpose to travel or as a co-captain, a case that this work could be extended to investigate.

The second constraint, (2) maintains that captains must not fly more than 60 hours in a 7-day period. Since the scope here is to a short period, this is easily done. The third constraint requires the use of indicators to signify the day. This lets us ensure that pilots do not fly more than 8 hours in a day. These two constraints can be found in the FAA pilot regulations. (FAA)

The fourth variable constraint requires that all of the decision variables are binary. A 1 reflects that a pilot is schedules, a 0 reflects the opposite. This is a natural way to represent this problem and can be useful in communicating the results.

VI. Other Formulation Considerations

The simpler versions of this case can be solved by hand, and we run the risk of over engineering in such instances. We would expect an optimal solution to resemble if-else structure; where we simply select the pilot with the lowest rate, check their availability, as per the FAA guidelines, and assign them if possible. The is depicted in Figure 1.

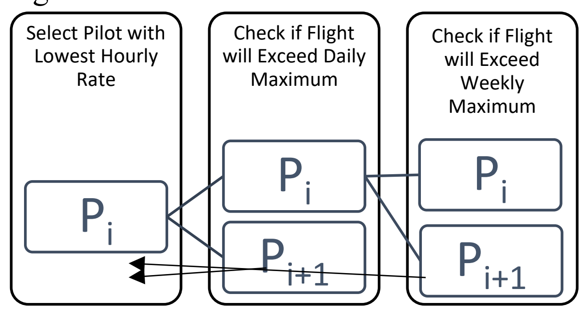

Figure 1: A figure depicting an assignment algorithm.

Another way we could formulate an assignment algorithm is by considering the pilots as a set, taking the subset of eligible pilots, and taking the one with the minimum rate. This results in a bridge between this algorithmic approach and the linear programming approach already established.

It is reasonable to assume that the binary programming method will yield the same results of these two methods, since the optimal solution is found by all.

VII. Results and Discussion

Here is a sample list of flights and pilots.

FLIGHTS

NAME	Depart	Arrive	Time	Day
F1	0	6	6	1
F2	8	10	2	1
F3	16	24	8	1
F4	25	30	5	2
F5	30	32	2	2
F6	49	56	7	3

F7	61	65	4	3
F8	66	74	8	3
F9	75	82	7	4
F10	88	93	5	4

PILOTS

NAME	Rate
P1	67
P2	65
P3	54
P4	41
P5	54

We take these two as inputs and generate the following assignment matrix:

DVS	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*	*F7*	*F8*	*F9*	*F10*
P1	0	0	0	0	0	0	0	0	0	0
P2	0	0	0	0	0	0	0	0	0	0
P3	0	0	1	0	1	0	1	0	0	1
P4	1	0	0	1	0	1	0	1	1	0
P5	0	1	0	0	0	0	0	0	0	0

We notice some interesting patters. First, we see that P1 and P2 are never required to fly. This is due to the fact that these two have the greatest hourly rates. As such, one possible application of this model is a business decision assistant: a business analyst could make the recommendation that contract of P1 and P2 be renegotiated on these grounds.

As we predicted we see that at every day, P4 is chosen first, and then either P3 or P4. This is again due to the rank ordering of the pilot's hourly rates. This implies that there is not exactly one optimal solution for cases like these. In practice we can look at this as an advantage: it allows us to accommodate things such as leave and delays of the pilots.

Of course, it is important to realize that this at scale can produce much finer results. Considering a post-optimality analysis is essential for understanding the function of this model. First, we can consider variations in the hourly wages of the pilots.

Using the algorithmic approach, we can understand that as the rates change, so does the favoring of the binary linear programming model. This allows us to gauge which pilots will be assigned first as well as which ones will not be assigned at all.

Second, as the time tables of the flight shift so will the cost function. Indeed, if it were to be the case that a pilot could only fly one flight a day due to FAA regulations, or some other various reason, the model presented here could capture that. Further, should the airline's database of pilots *not* be sufficiently large for the flights, our model would be able to be used to inform decision makers of this gap, and they would begin recruiting more pilots.

The results here relate to the original goals because we have a functioning binary linear programming model that allows us to map a list of flights and pilots to an assignment matrix. These results are similar to those found previously with the goal of taking requirements and regulations into account as we schedule crews, such as in (Oliver Weide, 2010) which discussed iterative solution methods. Indeed, we assert that this model can be constructed with an iterative approach as shown in Figure 1, however with binary linear programming we achieve a more robust model because it is more capable of being altered and added to. This would be a crucial feature for commercial use.

VIII. Conclusions and Future Research

Shown here is a binary linear programming method for scheduling pilots on flights. This has applications in areas of transportation science, operations research, and perhaps most so in commercial airline operations, which seek to maximize profit and minimize cost. The strength of a binary linear programming approach is the ease of adding and changing constraints. Certain regulations will vary as laws and requirements change. It is therefore advantageous to have a system readily in place that can accept these changing parameters.

Thus, we propose two strong areas to extend this research. The first is the simple addition of requirements. For example, night flying and day flying can be considered differently, which then takes into account changing time zones as well as progressions of sunrise/sun set.

Another interesting way to extend this research is the relaxation of the assumptions made here. In particular ways to represent and control for flights that have overlapping windows of time in the air would be useful for airlines to have. This would allow then for the model to ensure that there are no instances in which a pilot is assigned to a flight that departs before the current flight ends. This would require careful consideration of the variables to mitigate computational exploding that comes with temporal analysis using binary linear programming.

These extensions will not only develop methods of binary linear programming discussed here, but provide a more useful and applicable product to airlines for commercial implementation.

IX. References

Bowman, E. (1956). Production Scheduling by the Transportation Method of Lineat Programming. *Operations Research*, 100-103.

Brigitte Jaumard, F. S. (1998). A generalized linear programming model for nurse scheduling. *Europoen Journal of Operational Research*, 1-18.

FAA. (n.d.). § 91.1059 Flight time limitations and rest requirements: One or two pilot crews.

J. E. Beasley, M. K. (2000). Scheduling Aircraft Landings—The Static Case. *Transportation Science*, 133-238.

Jean-Francois Cordeau, G. S. (2001). Benders decomposition for simultaneous aircraft routing and crew scheduling. *Transportation Science*.

Julia L. Higle, S. S. (1996). *Stochastic Decomposition: A Statistical Method for Large Scale Stochastic Linear Programming.* Dordrecht: Kluwer Academic Publishers.

Kin Cheon Sou, J. W. (2011). Scheduling smart home appliances using mixed integer linear programming. *2011 50th IEEE Conference on Decision and Control and European Control Conference*.

Oliver Weide, D. R. (2010). An iterative approach to robust and integrated aircraft routing and crew scheduling.

Computers & Operations Research,
833-844.